AMATYC

STUDENT MATHEMATICS LEAGUE

TEST #1

INSTRUCTIONS FOR ANSWER SHEET

TO THE STUDENT:

You have one hour to take this test. Scratch paper, which will be supplied, is allowed. Also allowed are calculators, including graphics calculators, provided they do not have a typewriter-like keyboard (such as, for example, the TI-92) or a disk drive. Pdas are not allowed. **However, no books, mathematical tables, computers, or questions are permitted during the test.**

You are NOT expected to be able to answer all the questions; do not become discouraged at the difficulty of the questions. Do not waste too much time on one question, but return to it after you finish the others.

Each question has a 2 point value for a correct answer and a penalty of -1/2 point for each wrong answer. There is no deduction for unanswered questions or for wrong answers to questions which are not multiple choice. Do not make any random guesses, but do answer a question if you can eliminate some of the choices.

Put the letter for the best response to each question you answer in the first column on the answer sheet provided.

Are there any questions?

Do not begin until you are told to do so.

1. If L has equation $ax + by = c$, M is its reflection across the y-axis, and N is its reflection across the x-axis, which of the following must be true about M and N for all nonzero choices of a, b, and c?

A. the x-intercepts are equal B. the y-intercepts are equal C. the slopes are equal
D. the slopes are opposite E. the slopes are reciprocals

2. A collection of coins is made up of an equal number of pennies, nickels, dimes, and quarters. What is the largest possible value of the collection which is less than \$2?

A. \$1.64 B. \$1.78 C. \$1.86 D. \$1.89 E. \$1.99

3. When the polynomial P(x) is divided by $(x-2)^2$, the remainder is $3x - 3$. What is the remainder when $(x-1)P(x)$ is divided by $(x-1)(x-2)^2$?

A. $3x - 3$ B. $3x^2 - 6x + 3$ C. 3 D. $x - 1$ E. $x - 2$

4. If $f(x) = 3x - 2$, find $f(f(f(3)))$. A. 19 B. 55 C. 75 D. 107 E. 163

5. What is the remainder when $x^3 - 2x^2 + 4$ is divided by $x + 2$?

A. -12 B. 0 C. 4 D. 6 E. 12

6. Let p be a prime number and k an integer such that $x^2 + kx + p = 0$ has two positive integer solutions. What is the value of $k + p$?

A. 1 B. -1 C. 0 D. 2 E. -2

7. What is the least number of prime numbers (not necessarily different) that 3185 must be multiplied by so that the product is a perfect cube?

A. 1 B. 2 C. 3 D. 4 E. 5

8. Two adjacent faces of a three-dimensional rectangular box have areas 24 and 36. If the length, width, and height of the box are all integers, how many different volumes are possible for the box?

A. 2 B. 3 C. 4 D. 5 E. 6

9. $(\tan t - \sin t \cos t)/(\tan t) =$

A. $\sin t$ B. $\cos t$ C. $\sin^2 t$ D. $\cos^2 t$ E. 1

10. The counting numbers are written in the pattern at the right. Find the middle number of the 40^{th} row.

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                    1
                2   3   4
            5   6   7   8   9
        10  11  12  13  14  15  16
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A. 1561 B. 1641 C. 1559 D. 1639 E. 1483

11. The solution set of $x^2 - 3x - 18 \geq 0$ is a subset of the solution set of which of the following inequalities?

A. $x^2 - x - 20 \geq 0$ B. $(x - 4)/(x + 3) \geq 0$ C. $x^2 - 8x + 14 \geq 0$

D. both B and C E. all of A, B, and C

12. If $2a - 4b = 128b^3 - 16a^3$ and $a \neq 2b$, find $a^2 + 2ab + 4b^2$.

A. -1/8 B. -1/2 C. 1/2 D. 1/8 E. 2

13. Square ABCD is inscribed in circle O (that is, A, B, C, and D all lie on the circle) and its area is a. Square EFGH is inscribed in a semicircle of circle O (that is, E and F lie on a diameter and G and H lie on the circle). What is the area of square EFGH?

A. a/5 B. 2a/5 C. a/3 D. a/2 E. 3a/5

14. Consider all arrangements of the letters AMATYC with either the A's together or the A's on the ends. What fraction of all possible such arrangements satisfies these conditions?

A. 1/5 B. 2/15 C. 1/3 D. 2/5 E. 3/5

15. The year 2003 is prime, but its reversal, 3002, is not. In fact, 3002 is the product of exactly three different primes. Let N be the sum of these three primes. How many other positive integers are the products of exactly three different primes with this sum N?

A. 0 B. 1 C. 2 D. 3 E. 4

16. In a group of 30 students, 25 are taking math, 22 English, and 19 history. If the largest and smallest number who could be taking all three courses are M and m respectively, find M + m.

A. 17 B. 19 C. 22 D. 23 E. 25

17. A boat with an ill passenger is 7 1/2 mi north of a straight coastline which runs east and west. A hospital on the coast is 60 miles from the point on shore south of the boat. If the boat starts toward shore at 15 mph at the same time an ambulance leaves the hospital at 60 mph and meets the ambulance, what is the total distance (to the nearest 0.5 mile) traveled by the boat and the ambulance?

A. 60.5 B. 61 C. 61.5 D. 62 E. 62.5

18. If each letter in the equation $\sqrt{AMATYC} = MYM$ represents a different decimal digit, find T's value.

A. 3 B. 4 C. 5 D. 6 E. 7

19. If a, b, c, and d are nonzero numbers such that c and d are solutions of $x^2 + ax + b = 0$ and a and b are solutions of $x^2 + cx + d = 0$, find $a + b + c + d$.

A. -2 B. -1 C. 0 D. 1 E. 2

20. Al and Bob are at opposite ends of a diameter of a silo in the shape of a tall right circular cylinder with radius 150 ft. Al is due west of Bob. Al begins walking along the edge of the silo at 6 ft per second at the same moment that Bob begins to walk due east at the same speed. The value closest to the time in seconds when Al first can see Bob is

A. 46 B. 47 C. 48 D. 49 E. 50

PLEASE PRINT!

AMATYC STUDENT MATHEMATICS LEAGUE

NAME: KEY -- Nov 2003

COLLEGE: _____

ROUND: [1] 2

	Student's Responses	Local Corrector
1	**C**	
2	**A**	
3	**B**	
4	**B**	
5	**A**	
6	**B**	
7	**E**	
8	**E**	
9	**C**	
10	**A**	
11	**C**	
12	**A**	
13	**B**	
14	**D**	
15	**C**	
16	**E**	
17	**E**	
18	**E**	
19	**A**	
20	**C**	

correct = _____

incorrect = _____

blank = _____

$\square$ = # correct $\times$ 2

$-$ $\square$ = # incorrect $\times \frac{1}{2}$

$\square$ = **score**