Solutions to Application Problems assigned in class (set 1) - *Revised* (I think they are correct now...)

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1) Find the area between $\sin x$ and $\cos x$ (one "cell"):

$$\int_{-3\pi/4}^{\pi/4} (\text{Cos}[x] - \text{Sin}[x])\, dx$$

$$2\sqrt{2}$$

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2) Find the volume of a pyramid with an equilateral triangle base of side length 2 and height 6:

$$\int_0^6 \frac{\sqrt{3}}{4} \left(\frac{y}{3}\right)^2 dy$$

$$2\sqrt{3}$$

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3) Find the volume of the solids of revolution:

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a. between $y = x^2$ and $y = x^4$, about $x = 7$:

$$2\pi \int_{-1}^{1} (7 - x)\,(x^2 - x^4)\, dx$$

$$\frac{56\pi}{15}$$

$$\pi \int_0^1 \left(\left(7 - \sqrt{y}\right)^2 - \left(7 - \sqrt[4]{y}\right)^2\right) dy + \pi \int_0^1 \left(\left(7 + \sqrt[4]{y}\right)^2 - \left(7 + \sqrt{y}\right)^2\right) dy$$

$$\frac{56\pi}{15}$$

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b. between $y = x^3$ and $y = 4x$, about about $y = b$ (I said $y = 20$ in the morning class and changed it to something else, which I don't remember, in the afternoon class; this formula is valid for any $b \geq 8$.

$$\pi \left(\int_{-2}^{0} \left((b - 4x)^2 - (b - x^3)^2\right) dx + \int_0^2 \left((b - x^3)^2 - (b - 4x)^2\right) dx\right)$$

$$16\, b\, \pi$$

$$2\pi \left(\int_{-8}^{0} \left(\frac{y}{4} - \sqrt[3]{y}\right)(b - y)\, dy + \int_0^8 \left(\sqrt[3]{y} - \frac{y}{4}\right)(b - y)\, dy\right)$$

c. between $y = \ln x$, $y = 0$, $x = 2$, about $x = -3$: (note: Log here means natural log here; you may approximate the integral numerically)

$$2\pi \int_{1.}^{2} (x + 3)\ (\text{Log}[x])\ dx$$

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11.2794
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$$\pi \int_{0.}^{\text{Log}[2]} \left(25 - (e^{y} + 3)^{2}\right) dy$$

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11.2794
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4) Find work:

a. Roll up a 40 ft chain 1/4 of the way, with a 20 lb bucket on the end (chain weighs 2 lb/ft)

$$\int_{30}^{40} 2\ (40 - y)\ dy + 10\ (30)\ (2) + 10\ (20)$$

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900
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b. Pump all the water out of a cone (with sharp end down) with radius 2 ft, height 8 ft

$$62.4\pi \int_{0}^{8} \left(\frac{y}{4}\right)^{2}\ (8 - y)\ dy$$

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4182.09
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c. Pump half the water out of a sphere of radius 5 ft (assume sphere is full)

$$62.4\pi \int_{0}^{5} \left(\sqrt{25 - y^{2}}\right)^{2}\ (5 - y)\ dy$$

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51050.9
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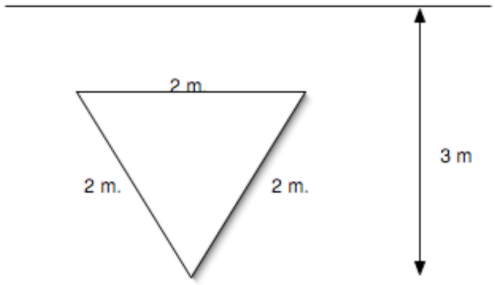
5) Find the fluid force ("hydrostatic force"):

a.



$$9.8\,(1000)\int_0^{\sqrt{3}}(3-y)\left(\frac{2y}{\sqrt{3}}\right)dy$$

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31322.3
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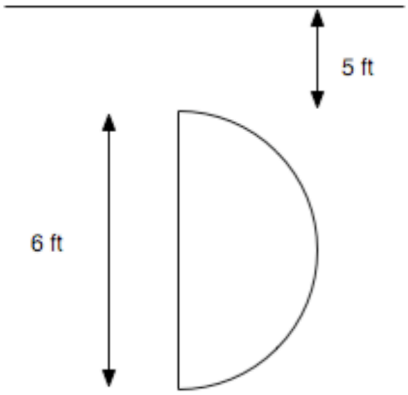
b.



$$62.4\int_{-3}^{3}(8-y)\sqrt{9-y^2}\,dy$$

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7057.27
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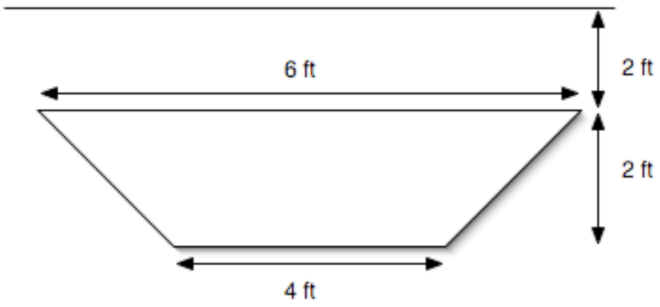
c.



$$62.4 \int_0^2 (4 - y) \left(2 \left(\frac{y}{2}\right) + 4\right) dy$$

1830.4