

If this doesn't work, try:

1

2

3

4

5

**Basic Known Integrals**

Derivatives of inverse trigonometric functions, ie

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(X)$$

$$\int \frac{dx}{1+x^2} = \arctan(X)$$

Or just basic trig functions, ie

$$\int \csc^2(x) dx = -\cot(x) + c$$

$$\int \sec^2(x) dx = \tan(x) + c$$

$$\int \csc(x) \cot(x) dx = -\csc(x) + c$$

$$\int \sec(x) \tan(x) dx = \sec(x) + c$$

And of course these,

$$\int \sin(x) dx = -\cos(x) + c$$

$$\int \cos(x) dx = \sin(x) + c$$

$$\int \tan(x) dx = \ln|\sec(x)| + c$$

$$\int \csc(x) dx = \ln|\csc(x) - \cot(x)| + c$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + c$$

$$\int \cot(x) dx = \ln|\sin(x)| + c$$

**Substitution**

There is not a whole to to say here other than:  
After you are sure there are **▲** no basic known integrals **▲** ;  
Try **SUBSTITUTION** and make sure **IT DOESN'T WORK** before you go and bust your **&#33!** on the following techniques!!!

**Integration by Parts (uv - ∫vdu)**

Use Integration by parts when your integral is:  
1. a product of two things (logarithmic, inverse trigonometric, algebraic, trigonometric, exponential,  
2. only one thing in cases of logarithmic or inverse trig

Pick your *U* and *DV* based on:

1. if you can't integrate one, make it your *U*
2. pick whatever gets simpler
3. if after using integration by parts, you're back to where you started, move it to the left

From Stewart's "Calculus" Solution's, the mnemonic device **LIATE**, a principle of precedence for choosinig *U*:  
Logarithmic, Inverse trig, Algebraic, Trigonometric, Exponential

**Trigonometric Substitution**

If you see something like this, consider substituting "w" with trig.

$$\left\{ \begin{array}{l} \sqrt{w^2 - a^2}; \frac{w}{a} = \sec \theta \\ \sqrt{a^2 - w^2}; \frac{w}{a} = \sin \theta \\ \sqrt{a^2 + w^2}; \frac{w}{a} = \tan \theta \end{array} \right.$$

If radical is missing  $(4+x^2)^3$  make your own  $(\sqrt{4+x^2})^6$

If quadratic  $\sqrt{x^2+4x-7}$  complete the square  $\sqrt{(x+2)^2-9}$

If radical is missing and it's quadratic, do both

**Trigonometric Integration (products of trig functions)**

Integrals in the form of  $\int \sin^m \theta \cos^n \theta dx$  If either m or n is odd; use the other as *U*. If both m and n are even, know the half angle or power reducing identity and use it. If both are odd, flip a coin.

$$\int \sin^3 \theta \cos^2 \theta d \theta = \int \sin^2 \theta \cos^2 \theta \sin \theta d \theta = \int (1 - \cos^2 \theta) \cos^2 \theta \sin \theta d \theta \text{ THEN } [w = \cos \theta]$$

Integrals in the form of  $\int \tan^m \theta \sec^n \theta dx$  If "n" is even, use  $\tan \theta$  as your *U*; If "n" is odd, integrate by parts; If "m" is odd, use  $\sec \theta$  ; if "m" is even, convert to  $\sec \theta$

**Partial Fractions**

If the numerator is the same degree or higher than the denominator, perform long division. Then integrate the resulting expression. Otherwise, consider the following steps

Factor the denominator, where each factor is:

- i) linear  $(ax+b)$ ,
- ii) an irreducible quadratic  $(ax^2+bx+c)$  or
- iii) a power of the form  $(ax+b)^n$  or  $(ax^2+bx+c)^n$

Perform partial fraction decomposition where each fraction is in the form of either:

- $F(x) = A/(ax+b)^k$  or
- $F(x) = Ax+B/(ax^2+bx+c)^k$

Solve for numerators using a system of equations or collecting like terms of *x*.